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# Gauge bosons and the $\mathrm{AdS}_3/\mathrm{LCFT}_2$ correspondence

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Abstract: We study the relationship between the gauge boson coupled to spin 2 operator and the singleton in three-dimensional anti-de Sitter space($\mathrm{AdS}_3$). The singleton can be expressed in terms of a pair of dipole ghost fields $A$ and $B$ which couple to $D$ and $C$ operators on the boundary of $\mathrm{AdS}_3$. These operators form the logarithmic conformal field theory($\mathrm{LCFT}_2$). Using the correlation function for logarithmic pair, we calculate the greybody factor for the singleton. In the low temperature limit of $\omega \gg T_{\pm}$, this is compared with the result of the bulk $\mathrm{AdS}_3$ calculation of the gauge boson. We find that the gauge boson cannot be realized as a model of the $\mathrm{AdS}_3/\mathrm{LCFT}_2$ correspondence.

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Recently the AdS/CFT correspondence has attracted much interest [1, 2, 3]. This means that the string/M theory on $\mathrm{AdS}_{d+1}$ is dual to the gauge theory of the $\mathrm{CFT}_d$ on its boundary. This was used to resolve many problems in black hole physics [4, 5, 7, 8]. For the test of the $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence one introduces a set of test fields $\{\Phi_i\}$ on $\mathrm{AdS}_3$ and their corresponding operators $\{\mathcal{O}^i\}$ on the boundary. For example, these are in the D1-D5 brane system : a free scalar ($\phi$) which couples to (1,1) operator; two fixed scalars ($\nu, \lambda$) to (2,2), (3,1), and (1,3) operators; two intermediate scalars ($\eta, \xi$) to (1,2) and (2,1) operators [9]. The relevant relation between these is given by [3]

$$e^{-S_{\rm eff}(\{\Phi_i\})} = \left\langle e^{\int_{\rm B} \Phi_{b,i}\mathcal{O}^i} \right\rangle . \qquad (1)$$

The expectation value $\langle \cdots \rangle$ is taken in the CFT with the boundary test field $\Phi_{b,i}$ as a source. Eq. (1) was widely used for calculations of the entropy, greybody factor (2-point function), 3- and 4-point functions. It was shown by studying exchange diagram with scalar and gauge fields in N=4 SUSY Yang-Mills from $\mathrm{AdS}_5$ that the 4-point function has logarithmic singularities [10]. More recently Kogan proposed that a dipole ghost pair $(A, B)$ can represent a singleton, which induces the 2-point correlation function for a logarithmic pair $(\mathcal{O}, \mathcal{O}')$ [11]. He argued that this is the origin of logarithmic singularities in the 4-point functions. This logarithmic pair with the normalization factor $c = 2(\Delta - 1)^2/\pi$ [7] has the 2-point correlation fucntions [12, 13]

$$\begin{aligned}
\langle \mathcal{O}(\mathbf{x})\mathcal{O}'(\mathbf{y})\rangle &= \frac{c}{|\mathbf{x}-\mathbf{y}|^{2\Delta}} \,, \\
\langle \mathcal{O}'(\mathbf{x})\mathcal{O}'(\mathbf{y})\rangle &= \frac{c}{|\mathbf{x}-\mathbf{y}|^{2\Delta}}\left[-2\ln|\mathbf{x}-\mathbf{y}| + \frac{1}{c}\frac{\partial c}{\partial \Delta}\right], \\
\langle \mathcal{O}(\mathbf{x})\mathcal{O}(\mathbf{y})\rangle &= 0 \,.
\end{aligned} \qquad (2)$$

Here $\Delta$ is a degenerate dimension $\Delta$ of $\mathcal{O}$ and $\mathcal{O}'$ and we note a crucial relation $\langle \mathcal{O}'(\mathbf{x})\mathcal{O}'(\mathbf{y})\rangle = \partial\langle \mathcal{O}(\mathbf{x})\mathcal{O}'(\mathbf{y})\rangle/\partial\Delta$. We will use this relation to calculate the greybody factor.

In the $\mathrm{AdS}_3/\mathrm{CFT}_2$ correspondence, there exists a puzzle of the missing states between $\mathrm{CFT}_2$ and supergravity [14]. The gauge bosons appear in the resolution of this puzzle. These are chiral primaries which correspond to the descendent of the identity operator in the $\mathrm{CFT}_2$ [6]. But on the supergravity side these are absent and thus may be considered as unphysical singletons on $\mathrm{AdS}_3$ [15]. In this sense, it is important to test the relationship between gauge boson and singleton. The authors in [16] found that these gauge bosons coupled to (2,0) and (0,2) operators on the boundary receive logarithmic corrections from a bulk $\mathrm{AdS}_3$ scattering calculation. Of course this was performed in the low temperature limit of $\omega \gg T_\pm$.

In this paper, we derive the greybody factor for the singleton. According to ref. [11], we wish to represent this with a pair of dipole ghost fields $(A, B)$ coupled to (1,1) operators $(D, C)$. This means that a gauge boson with spin 2 assumed to be expressed in terms of a pair of dipole ghost fields with spin 0. And then we calculate the two-point function of their operators in terms of the BTZ coordinates. Using this, we obtain the greybody factor. Finally we will compare this with the result of a bulk $AdS_3$ scattering in the low-temperature limit.

We start with the bulk $AdS_3$ action for a dipole pair $(A, B)$ [12, 11]

$$S_{\rm eff} = \int d^3x\sqrt{g}\left[\partial A\cdot\partial B - m^2AB - \frac{1}{2}B^2\right]. \tag{3}$$

At this stage it is not clear if this action comes from supergravity (string) theories. Rather, (3) takes a similar form of the Nakanishi-Lautrup formalism in the gauge theory [17]. In detail, (3) with $m^2 = 0$ and $A_\mu = \partial_\mu A$ leads to the gauge-fixing term as $S_{\rm GF} = -\int d^3x\sqrt{g}\left[B\partial_\mu A^\mu + \frac{\alpha}{2}B^2\right]$ with $\alpha = 1$. Here $B$ is the Nakanishi-Lautrup field. $A$ corresponds to $\sigma$ in [17] and leads to the negative-norm state. Its equations of motion are given by

$$\left(\nabla^2 + m^2\right)A + B = 0\,, \qquad \left(\nabla^2 + m^2\right)B = 0\,. \tag{4}$$

A solution to these can be found from a boundary-bulk Green function for $\langle AB\rangle$ with mass $m$ [18, 8]

$$\begin{aligned} &K_{AB}(r,u_+,u_-;u'_+,u'_-) = K_{BA}(r,u_+,u_-;u'_+,u'_-) = \\ &= N\left\{\frac{\pi^2T_+T_-}{\frac{r_+^2-r_-^2}{4r}\exp(\pi[T_+\delta u_+ + T_-\delta u_-]) + r\sinh(\pi T_+\delta u_+)\sinh(\pi T_-\delta u_-)}\right\}^\Delta, \end{aligned} \tag{5}$$

where we used the coordinates $(r, t, \phi)$ in the BTZ black hole with $R = 1$: mass $M = (r_+^2 - r_-^2)$, angular momentum $J = 2r_+r_-$, and left/right temperatures $T_\pm = (r_+ \pm r_-)/2\pi$ [19]. The Hawking temperature $T_H$ is defined by $2/T_H = (1/T_+ + 1/T_-)$. Here $u_\pm = \phi \pm t$, $\delta u_\pm = u_\pm - u'_\pm$, $\Delta(\Delta - 2) = m^2$. The normalization constant $N$ is introduced for convenience. To find $K_{AA}$ we have to solve the equation

$$\left(\nabla^2 + m^2\right)K_{AA} = -K_{AB}\,. \tag{6}$$

This is found by a trick as

$$K_{AA} = \frac{\partial K_{AB}}{\partial m^2} = \frac{1}{2(\Delta-1)}\frac{\partial K}{\partial\Delta}\,. \tag{7}$$

Then the solution is given by

$$\begin{aligned} A(r,u_+,u_-) = &\int du'_+du'_-\Big[K_{AB}(r,u_+,u_-;u'_+,u'_-)B_b(u'_+,u'_-) + \\ &\qquad + K_{AA}(r,u_+,u_-;u'_+,u'_-)A_b(u'_+,u'_-)\Big]\,, \\ B(r,u_+,u_-) = &\int du'_+du'_-K_{BA}(r,u_+,u_-;u'_+,u'_-)A_b(u'_+,u'_-)\,, \end{aligned} \tag{8}$$

where $A_b$ and $B_b$ are the boundary values of $A$ and $B$ respectively. Using (3) and (4), the effective action takes only the boundary form

$$S_{\rm eff}[A_b, B_b] = \lim_{r_s\to\infty} \frac{1}{2}\int_S du_+ du_- \sqrt{-h}\,\{A(\hat{n}\cdot\nabla)B + B(\hat{n}\cdot\nabla)A\}\,, \qquad (9)$$

where $S$ is a regularized surface at $r = r_s$ and $\hat{n}\cdot\nabla = r(\partial/\partial r)$. $h_{\mu\nu}$ is an induced boundary metric with $\mathrm{diag}(-r^2, r^2)$ and thus $\sqrt{-h} = r^2$. Considering the boundary behavior of $A$ and $B$ as

$$A(B)|_{r\to\infty} \sim r^{-2+\Delta} A_b(B_b)\,, \qquad (10)$$

one finds

$$\begin{aligned} S_{\rm eff}[A_b, B_b] = & -\frac{\Delta N}{2}\int du_+ du_- du'_+ du'_- \left[\frac{\pi T_+}{\sinh(\pi T_+ \delta u_+)}\right]^{\Delta}\left[\frac{\pi T_-}{\sinh(\pi T_- \delta u_-)}\right]^{\Delta}\times \\ & \times\Bigg[2A_b(u_+,u_-)B_b(u'_+,u'_-) + \frac{A_b(u_+,u_-)A_b(u'_+,u'_-)}{2(\Delta-1)}\times \\ & \times\left\{\frac{1}{N}\frac{\partial N}{\partial\Delta} - \ln\left(\frac{\sinh(\pi T_+\delta u_+)}{\pi T_+}\cdot\frac{\sinh(\pi T_-\delta u_-)}{\pi T_-}\right)\right\}\Bigg]. \qquad (11)\end{aligned}$$

With (1), one can derive the two-point functions for conformal operators $C$ and $D$ as

$$\langle C(u_+,u_-)C(0)\rangle = -\frac{\delta^2 S[A_b,B_b]}{\delta B_b(u_+,u_-)\delta B_b(0)} = 0, \qquad (12)$$

$$\begin{aligned}\alpha\langle C(u_+,u_-)D(0)\rangle &= -\frac{\delta^2 S[A_b,B_b]}{\delta B_b(u_+,u_-)\delta A_b(0)} \\ &= \Delta N\left[\frac{\pi T_+}{\sinh(\pi T_+ u_+)}\right]^{\Delta}\left[\frac{\pi T_-}{\sinh(\pi T_- u_-)}\right]^{\Delta}, \qquad (13)\end{aligned}$$

$$\begin{aligned}\beta\langle D(u_+,u_-)D(0)\rangle &= -\frac{\delta^2 S[A_b,B_b]}{\delta A_b(u_+,u_-)\delta A_b(0)} \\ &= \frac{N\Delta}{4(\Delta-1)}\left[\frac{\pi T_+}{\sinh(\pi T_+ u_+)}\right]^{\Delta}\left[\frac{\pi T_-}{\sinh(\pi T_- u_-)}\right]^{\Delta}\times \\ &\quad\times\left[\frac{1}{N}\frac{\partial N}{\partial\Delta} - \ln\left\{\frac{\sinh(\pi T_+ u_+)}{\pi T_+}\cdot\frac{\sinh(\pi T_- u_-)}{\pi T_-}\right\}\right], \qquad (14)\end{aligned}$$

where $\alpha, N$, and $\beta$ are chosen to recover (2) in the low temperature limit of $T_\pm \to 0$. Then one finds $\alpha = \Delta$, $N = c$, $\beta = \Delta/4(\Delta-1)$. We are now in a position to calculate the greybody factor using the above correlation functions [20]. The greybody factor for $\langle AB\rangle$ is calculated by the boundary CFT as [21, 7]

$$\begin{aligned}\sigma^{AB}_{\rm abs} &= \frac{\pi}{\omega}\int dt\int_{-\infty}^{\infty} d\phi e^{-i\omega t + ip\phi}\left[\langle C(t-i\epsilon,\phi)D(0)\rangle - \langle C(t+i\epsilon,\phi)D(0)\rangle\right] = \\ &= \frac{2(\Delta-1)^2(2\pi T_+ R)^{\Delta-1}(2\pi T_- R)^{\Delta-1}\sinh(\omega/2T_H)}{\omega\Gamma^2(\Delta)}\times \\ &\quad\times\left|\Gamma\left(\frac{\Delta}{2} + i\frac{\omega}{4\pi T_+}\right)\Gamma\left(\frac{\Delta}{2} + i\frac{\omega}{4\pi T_-}\right)\right|^2, \qquad (15)\end{aligned}$$



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where $R^{2(\Delta-1)}$ has been switched on $\langle CD\rangle$, to recover a complete form of the greybody factor. Here the original integral region of $0 \le \phi \le 2\pi$ is changed into $-\infty \le \phi \le \infty$ to accommodate the periodic nature of $u_\pm \sim u_\pm + 2\pi n$ for the BTZ black hole [8]. For $\Delta = 2(m^2 = 0)$, (15) takes exactly the same form of the greybody factor for a massless minimally coupled scalar ($\nabla^2\Phi = 0$) [22]

$$\sigma^{AB}_{\rm abs} = \pi^2 \omega R^2 \frac{e^{\omega/T_H} - 1}{\left(e^{\omega/2T_+} - 1\right)\left(e^{\omega/2T_-} - 1\right)}\,. \tag{16}$$

In the low energy limit of $\omega \ll T_\pm$ one finds $\sigma^{AB}_{\rm abs}|_{\omega<T_\pm} = 2\pi r_+ = \mathcal{A}_H$, while in the low temperature limit of $\omega \gg T_\pm$ it takes $\sigma^{AB}_{\rm abs}|_{\omega>T_\pm} = \pi^2\omega R^2$.

It seems to be difficult to calculate the greybody factor $\sigma^{AA}_{\rm abs}$ directly, because of the logarithmic singularities in (14). Instead, we use the relation of $\langle D(u_+,u_-)D(0)\rangle = \frac{\partial}{\partial\Delta}\langle D(u_+,u_-)C(0)\rangle$ and thus expect to find $\sigma^{AA}_{\rm abs} \simeq \partial\sigma^{AB}_{\rm abs}/\partial\Delta$. In this calculation we have to use the relation for the gamma function as

$$\frac{\partial\Gamma(z)}{\partial\Delta} = \frac{\partial z}{\partial\Delta}\frac{\partial\Gamma(z)}{\partial z} = \frac{\partial z}{\partial\Delta}\Gamma(z)\psi(z)\,, \tag{17}$$

where $\psi(z) = \partial \ln\Gamma(z)/\partial z$ is a digamma function. Finally we obtain the greybody factor for $\langle AA\rangle$ by using the boundary $\mathrm{LCFT}_2$ correlator $\langle DD\rangle$ as

$$\begin{aligned}
\sigma^{AA}_{\rm abs} &= \frac{\pi}{\omega}\int dt \int_{-\infty}^{\infty} d\phi e^{-i\omega t + ip\phi}\left[\langle D(t-i\epsilon,\phi)D(0)\rangle - \langle D(t+i\epsilon,\phi)D(0)\rangle\right] \simeq \\
&\simeq \frac{2(\Delta-1)^2(2\pi T_+R)^{\Delta-1}(2\pi T_-R)^{\Delta-1}\sinh(\frac{\omega}{2T_H})}{\omega\Gamma^2(\Delta)} \times \\
&\times\left[\frac{2}{\Delta-1} + \ln(2\pi T_+R) + \ln(2\pi T_-R) - 2\psi(\Delta) + \frac{1}{2}\psi\left(\frac{\Delta}{2} + i\frac{\omega}{4\pi T_+}\right) + \right.\\
&\left. + \frac{1}{2}\left\{\psi(\frac{\Delta}{2} - i\frac{\omega}{4\pi T_+}) + \psi(\frac{\Delta}{2} + i\frac{\omega}{4\pi T_-}) + \psi(\frac{\Delta}{2} - i\frac{\omega}{4\pi T_-})\right\}\right] \times \\
&\times \left|\Gamma\left(\frac{\Delta}{2} + i\frac{\omega}{4\pi T_+}\right)\Gamma\left(\frac{\Delta}{2} + i\frac{\omega}{4\pi T_-}\right)\right|^2 .
\end{aligned} \tag{18}$$

As far as we know, this is the first result for a dipole ghost pair (singleton). Now let us compare this with the result of a gauge boson with $s = 2$ from a bulk $\mathrm{AdS}_3$ scattering [16]

$$\sigma^{\rm gb}_{\rm abs} = \pi^2\omega R^2 s^2\left[1 + \omega R s \ln(2\omega R s)\right]. \tag{19}$$

For this purpose, we take the low temperature limit of $\omega \gg T_\pm$ and $\Delta = 2$ on $\sigma^{AA}_{\rm abs}$. In this case the digamma function $\psi$ can take an asymptotic form as [23]

$$\Re\psi(1+iy) = \Re\psi(1-iy) \simeq \ln y + \frac{1}{12y^2} + \frac{1}{120y^4} + \cdots\,. \tag{20}$$

In the low temperature limit, $\sigma^{AA}_{\rm abs}$ takes the form

$$\sigma^{AA}_{\rm abs} = \pi^2\omega R^2\left[1 + 2\ln(\omega R) + c'\right], \tag{21}$$

where $c' = 2\gamma - 1 - 2\ln 2$ with the Euler's constant $\gamma = 0.5772$. At the first sight, it seems that the (21) takes a similar form as in (19). In (19) the logarithmic term is multiplied by $\omega R$ and thus it is a subleading-order. However, in (21) one cannot find such a prefactor and $2\ln(\omega R)$ is regarded as the leading low energy behavior. In this sense, we argue that there is no agreement between the $\mathrm{AdS}_3$ calculation of gauge field and the LCFT result of a singleton. Further, the non-logarithmic terms in (21) do not agree with that of (19). If the spin 2 gauge boson is truely represented by a pair of dipole ghost fields on $\mathrm{AdS}_3$, from the AdS/LCFT correspondence (19) should agree with (21) even in the low-temperatue limit. Hence we conclude that the gauge boson with spin 2 has nothing to do with the AdS/LCFT correspondence.

On the other hand one finds the logarithmic operators in (14), which may induce the unitarity problem. Here we may resolve this problem. It is noted that these logarithmic terms originate from the unphysical dipole ghost fields $(A, B)$. As was shown in [17], this pair $(A, B)$ is turned into the zero-norm state by the Goldstone dipole mechanism in Minkowski spacetime. We suggest that the boundary logarithmic terms come from the negative-norm state of $A$. In order to remove the negative-norm state, we impose the subsidiary condition as $B^{+}(x)|0\rangle_{\rm phys} = 0$. Then the physical space ($|0\rangle_{\rm phys}$) will not include any $A$-particle state. This corresponds to the dipole mechanism to cancel the negative-norm state. Similarily, we expect that in the boundary $\mathrm{CFT}_2$ of $\mathrm{AdS}_3$, the theory can be managed to be unitary by choosing an appropriate subsidiary condition.

In conclusion, we derive the new greybody factor for a singleton from the $\mathrm{LCFT}_2$ correlator $\langle DD\rangle$ which corresponds to the derivative of the $\mathrm{CFT}_2$ correlator $\langle DC\rangle$ with respect to the weight $\Delta$. In the low temperature limit the bulk $\mathrm{AdS}_3$ result $\sigma^{\rm gb}_{\rm abs}$ does not lead to $\sigma^{AA}_{\rm abs}$ of the $\mathrm{LCFT}_2$ correctly. This means that the spin 2 gauge boson cannot be expressed in terms of a pair of dipole ghost fields $(A, B)$.

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